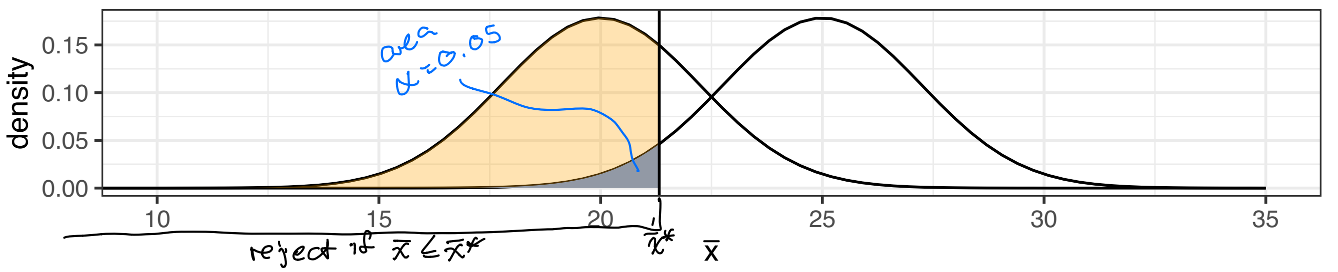
Warm Up: Motivation for the Neyman-Pearson Lemma

- Data Model: $X_1, \dots, X_5 \overset{\text{i.i.d.}}{\sim} \text{Normal}(\theta, 5^2)$
- Let's consider a test of the hypotheses $H_0 : \theta = 25$ vs. $H_A : \theta = 20$
- If H_0 is correct, then $\bar{X} \sim \text{Normal}(25, 5^2/5)$. If H_A is correct, then $\bar{X} \sim \text{Normal}(20, 5^2/5)$

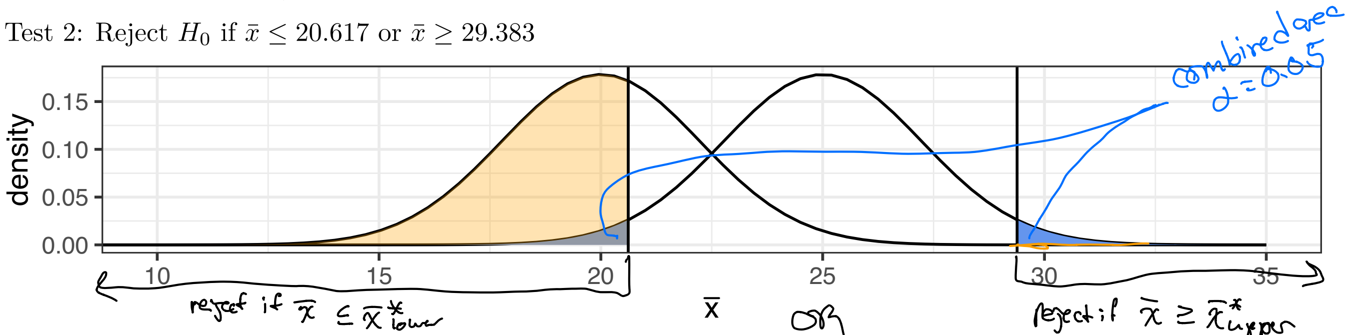
1. The following pictures can be used to illustrate 3 different tests based on the sampling distribution of $\bar{X}$, all with P(Type I Error | H_0 true) = 0.05. For each test,

- **Shade in the area corresponding to the probability of a Type I Error** (blue) $P(\bar{X} \le \bar{x}^* | H_0 \text{ correct})$
- **Shade in the area corresponding to the power of the test** (orange) $P(\bar{X} \le \bar{x}^* | H_A \text{ correct})$

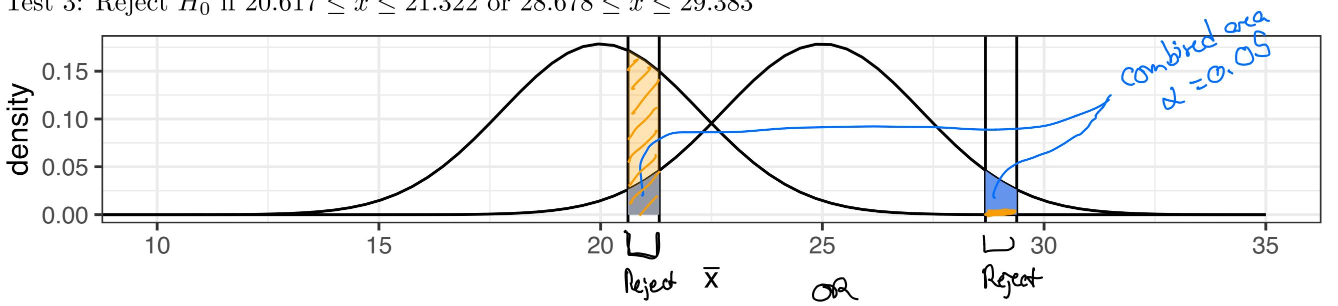
Test 1: Reject H_0 if $\bar{x} \le 21.322$ (This is the likelihood ratio test: for values of $\bar{x} \le 21.322$, the p-value is ≤ 0.05.)



Test 2: Reject H_0 if $\bar{x} \le 20.617$ or $\bar{x} \ge 29.383$



Test 3: Reject H_0 if $20.617 \le \bar{x} \le 21.322$ or $28.678 \le \bar{x} \le 29.383$



2. Which of the tests above has the highest power?

The likelihood ratio test.

3. For the likelihood ratio test, write down how you would calculate the probability of making a Type I Error and the power of the test as suitable integrals of either $f_{\bar{X}|\theta}(\bar{x}|20)$ or $f_{\bar{X}|\theta}(\bar{x}|25)$. (You will have 1 integral for the probability of a Type I Error and a second for the power of the test.)

$P(\text{Type I Error}|H_0 \text{ correct}) = \int_{-\infty}^{21.322} f_{\bar{X}|\theta}(\bar{x}|25) d\bar{x}$

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